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Optical Fresnel Transform as a Correspondence of the SU(1,1) Squeezing Operator Composed of Quadratic Combination of Canonical Operators and the Entangled State

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We study optical Fresnel transforms by finding the appropriate quantum mechanical SU(1,1) squeezing operators which are composed of quadratic combination of canonical operators. In one-mode case, the squeezing operator's matrix element in the coordinate basis is just the kernel of one-dimensional generalized Fresnel transform (GFT); while in two-mode case, the matrix element of the squeezing operator in the entangled state basis leads to the two-dimensional GFT kernel. The work links optical transforms in wave optics to generalized squeezing transforms in quantum optics.

KEY WORDS: Fresnel transform; SU(1,1) squeezing operator; the entangled state.

1. INTRODUCTION

In wave optics theory, the Fresnel integral is frequently used to describe beam propagation in paraxial approximation and Fresnel diffraction of light. The generalized Fresnel transform (GFT) for an arbitrary function $f(x_1)$ is defined as (Alieva and Agullo–Lopez, 1995; James and Agarwal, 1996)

$$\left[\mathcal{R}^M f(x_1)\right](x_2) = \int_{-\infty}^{\infty} \mathcal{K}^M (x_2, x_1)\, f(x_1)\, dx_1 \tag{1}$$

with the transform kernel

$$\mathcal{K}^M (x_2, x_1) = \frac{1}{\sqrt{2\pi i B}} \exp\left[\frac{i}{2B}\left(Ax_1^2 - 2x_2 x_1 + Dx_2^2\right)\right], \tag{2}$$

which is parameterized by a real matrix $M = [A, B; C, D]$ whose determinant $AD - BC = 1$. Since GFT is related to a wide class of optical transforms:

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Fourier transform, optical Wigner transform, wavelet transform and fractional Fourier transform (Namias, 1980; Mendlovic and Ozakatas, 1993), etc, its various properties and applications have brought great interests of physicists recently.

In this work, we shall study optical Fresnel transform by finding two appropriate quantum mechanical SU(1,1) squeezing operators, which are composed of quadratic combination of canonical operators in both one-mode and two-mode cases, respectively. The reason why we choose the quadratic combination of canonical operators lies in two aspects: (1) the corresponding optical processes for GFT can be analyzed more clearly and physically; (2) Generalized Fresnel Transform in wave optics can also be studied in terms of quantum optics operator transform method. The two kinds of squeezing operators we find in this paper have remarkable property: for one-mode SU(1,1) squeezing operator, its matrix elements in the coordinate representation is just the kernel of one-dimensional GFT; while for two-mode SU(1,1) squeezing operator, its matrix element in the entangled state basis leads to the two-dimensional GFT kernel (the entangled state of continuum variables has beeen constructed based on the quantum entanglement of Einstein-Podolsky-Rosen). The quadratic combination of canonical operators in two-mode case can be analyzed very conveniently in the entangled state representation. Important properties of GFT can be directly obtained via SU(1,1) squeezing operator approach. Thus a "bridge" linking optical transforms in wave optics to representation transform in quantum optics is established.

The work is arranged as follows. In Section 2, we introduce one-mode SU(1,1) squeezing operator composed by quadratic combination of canonical operators. Section 3 is devoted to showing the advantages of SU(1,1) squeezing operator. In Section 4, we will propose SU(1,1) squeezing operator in two-mode case. As an application of introducing SU(1,1) squeezing operator, we show in Section 5 that the parameter matrix corresponding to the scaling transform of two-dimensional GFT can be identified, and then the scaling law of GFT is derived in terms of two-mode SU(1,1) squeezing operator and the entangled state.

2. ONE-MODE SU(1,1) SQUEEZING OPERATOR BY QUADRATIC COMBINATION OF CANONICAL OPERATORS

We now search for one-mode SU(1,1) squeezing operator, the operator counterpart of one-dimensional GFT. SU(1,1) squeezing operator is related to GFT in such a way that by taking the matrix element of SU(1,1) squeezing operator in coordinate representation will yield the transform kernel of GFT. Enlightened by the optical Wigner transform theory, and its quantum mechanical correspondence (Fan, 2003), we know that the thin lens transform operator is $\exp\left(i\mu X^2/2\right)$, the Fresnel diffraction transform operator is $\exp\left(-i\nu P^2/2\right)$, and the optical scaling transform corresponding to the operator $\exp\left(-\frac{i}{2}\left(XP+PX\right)\ln\lambda\right)$, all the operators X^2, P^2 and $(XP + PX)$ are named quadratic canonical operators, because the commutative relation $[X, P] = i$ is canonical. These operators may constitute a SU(1,1) squeezing operator, and after some tries we finally find that the one-mode SU(1,1) squeezing operator reads

$$F_1(A, B, C) = \exp\left(\frac{iC}{2A}X^2\right)\exp\left(-\frac{i}{2}(XP + PX)\ln A\right)\exp\left(-\frac{iB}{2A}P^2\right),$$
$$AD - BC = 1. \tag{3}$$

To confirm this, we calculate matrix element of F_1 sandwiching between the coordinate basis $\langle x|$ and the momentum basis $|p\rangle$,

$$\begin{aligned}\langle x| F_1 |p\rangle &= \exp\left(\frac{iC}{2A}x^2\right)\langle x|\exp\left(-\frac{i}{2}(XP + PX)\ln A\right)|p\rangle\exp\left(-\frac{iB}{2A}p^2\right)\\ &= \frac{1}{\sqrt{2\pi A}}\exp\left(\frac{iC}{2A}x^2 - \frac{iB}{2A}p^2 + \frac{ipx}{A}\right),\end{aligned} \tag{4}$$

where we have used the squeezing property

$$\exp\left(-\frac{i}{2}(XP + PX)\ln A\right)|p\rangle = \frac{1}{\sqrt{A}}|p/A\rangle. \tag{5}$$

Using the completeness relation of $|p\rangle$, and $AD - BC = 1$, we see

$$\begin{aligned}\langle x_2| F_1 |x_1\rangle &= \int_{-\infty}^{\infty} dp\, \langle x_2| F_1 |p\rangle \langle p| x_1\rangle\\ &= \exp\left(\frac{iC}{2A}x_2^2\right)\frac{1}{2\pi\sqrt{A}}\int_{-\infty}^{\infty} dp \exp\left[-\frac{iB}{2A}p^2 + \frac{ipx_2}{A} - ipx_1\right]\\ &= \frac{1}{\sqrt{2\pi iB}}\exp\left[\frac{i}{2B}\left(Ax_1^2 - 2x_2x_1 + Dx_2^2\right)\right] \equiv \mathcal{K}^M(x_2, x_1),\end{aligned} \tag{6}$$

Thus $F_1(A, B, C)$ is really the expected SU(1,1) squeezing operator. It is easily seen

$$F_1^{-1}\begin{pmatrix} X \\ P \end{pmatrix} F_1 = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} X \\ P \end{pmatrix}. \tag{7}$$

3. ADVANTAGES OF INTRODUCING SU(1,1) SQUEEZING OPERATOR

Now the multiplication rule of GFT can be seen directly by virtue of SU(1,1) squeezing operator algebra. Using (7) we have

$$F_1'^{-1}\left(A', B', C'\right) F_1^{-1}(A, B, C)\begin{pmatrix} X \\ P \end{pmatrix} F_1(A, B, C) F_1'\left(A', B', C'\right)$$

$$= \begin{pmatrix} A'' & B'' \\ C'' & D'' \end{pmatrix} \begin{pmatrix} X \\ P \end{pmatrix}, \tag{8}$$

which implies

$$F_1''\left(A'', B'', C''\right) = F_1(A, B, C)\, F_1'\left(A', B', C'\right),$$
$$\begin{pmatrix} A'' & B'' \\ C'' & D'' \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{9}$$

Besides, it follows from (6) and (9) that

$$\langle x_3 | F_1\left(A'', B'', C''\right) | x_1 \rangle = \langle x_3 | F_1'\left(A', B', C'\right) \int_{-\infty}^{\infty} dx_2 \, |x_2\rangle \langle x_2 | F_1(A, B, C) | x_1 \rangle, \tag{10}$$

which indicates the group multiplication rule of GFT

$$\int_{-\infty}^{\infty} dx_2 \mathcal{K}^{M'}(x_3, x_2)\, \mathcal{K}^{M}(x_2, x_1) = \mathcal{K}^{M''}(x_3, x_1). \tag{11}$$

By observing the three separate exponentials of SU(1,1) squeezing operator in (3) we realize that it just one-to-one corresponds to the decomposition,

$$\begin{pmatrix} 1 & 0 \\ C/A & 1 \end{pmatrix} \begin{pmatrix} A & 0 \\ 0 & 1/A \end{pmatrix} \begin{pmatrix} 1 & B/A \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{12}$$

which shows that an arbitrary GFT can always be implemented by such an optical setup: let the light first go through a Fresnel diffraction, then followed by a scaling transform, and finally a thin lens transform. To this end, one can see that the reason of using the quadratic combination of canonical operators to express SU(1,1) squeezing operator is: the corresponding optical processes for GFT can be analyzed more clearly and physically.

Another advantage of introducing SU(1,1) squeezing operator lies in that now we can analyze how optical field state in quantum optics theory will be transformed under the Fresnel transformation, for example, the vacuum state undergoes the transform

$$\begin{aligned} & F_1(A, B, C)\, |0\rangle \\ &= \int\int_{-\infty}^{\infty} dx_1 dx_2 \, |x_2\rangle \langle x_2 | F_1(A, B, C) | x_1 \rangle \langle x_1 | \, 0 \rangle \\ &= \frac{1}{\pi^{1/2}} \int\int_{-\infty}^{\infty} dx_1 dx_2 \mathcal{K}_1^M(x_2, x_1) \exp\left[-\frac{x_1^2}{2} - \frac{x_2^2}{2} + \sqrt{2} a^\dagger x_2 - \frac{a_2^{\dagger 2}}{2} \right] |0\rangle \\ &= \sqrt{\frac{2}{A + D + i(B - C)}} \exp\left[\frac{A - D + i(B + C)}{2[A + D + i(B - C)]} a^{\dagger 2} \right] |0\rangle, \end{aligned} \tag{13}$$

which is a generalized squeezed state. In Mandel and Wolf's quantum optics book (1995), it is discussed that when a light beam enters into one port of an optical beamsplitter, the quantum effect of a vacuum state on the another part of the beamsplitter must be taken into account, so we think that the vacuum state might have effect on other optical instruments during Fresnel transformation in the context of quantum optics.

4. SU(1,1) SQUEEZING OPERATOR IN TWO-MODE CASE AND ENTANGLED STATE REPRESENTATION

Now we extend the one-mode SU(1,1) squeezing operator to a two-mode case. Note that the quadratic combinations $\frac{1}{4}\left((X_1 - X_2)^2 + (P_1 + P_2)^2\right)$, $\frac{1}{4}\left((X_1 + X_2)^2 + (P_1 - P_2)^2\right)$ and $i\,(X_1 P_2 + X_2 P_1)$ of the four canonical operators $(X_1,\ X_2;\ P_1,\ P_2)$ obey the commutative relations

$$\left[\frac{1}{4}\left((X_1 - X_2)^2 + (P_1 + P_2)^2\right), \frac{1}{4}\left((X_1 + X_2)^2 + (P_1 - P_2)^2\right)\right] = -\frac{i}{2}\,(X_1 P_2 + X_2 P_1) \tag{14}$$

and

$$\left[-\frac{i}{2}\,(X_1 P_2 + X_2 P_1), \frac{1}{4}\left((X_1 - X_2)^2 + (P_1 + P_2)^2\right)\right] = \frac{1}{4}\left[(X_1 - X_2)^2 + (P_1 + P_2)^2\right],$$
$$\left[-\frac{i}{2}\,(X_1 P_2 + X_2 P_1), \frac{1}{4}\left((X_1 + X_2)^2 + (P_1 - P_2)^2\right)\right] = -\frac{1}{4}\left[(X_1 + X_2)^2 + (P_1 - P_2)^2\right], \tag{15}$$

which shows a SU(1,1) Lie algebra structure, this structure is also complied by $X^2/2$, $P^2/2$ and $-i\,(XP + PX)/2$ that have been used in composing $F_1\,(A, B, C)$, thus we introduce two-mode SU(1,1) squeezing operator by

$$F_2\,(A, B, C) = \exp\left(\frac{iC}{2A}\left[(X_1 - X_2)^2 + (P_1 + P_2)^2\right]\right)\exp\left(i\,(X_1 P_2 + X_2 P_1)\ln A\right) \times \exp\left(-\frac{iB}{2A}\left[(X_1 + X_2)^2 + (P_1 - P_2)^2\right]\right). \tag{16}$$

To see how it is related to two-mode GFT, we introduce two mutually conjugate entangled states $|\eta\rangle$ and $|\zeta\rangle$ (Fan and Klauder, 1994; Fan and Xiong, 1995; Fan and Lu, 2003; Fan, 2003, 2004; Fan et al., 2003, 2004; Fan and Fu, in press; Hong-Yi

Fan and Jiang, 2004)

$$|\eta\rangle = \exp\left[-\frac{|\eta|^2}{2} + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger\right]|00\rangle\,,\ \eta = \eta_1 + i\eta_2,$$

$$|\zeta\rangle = \exp\left[-\frac{|\zeta|^2}{2} + \zeta a_1^\dagger + \zeta^* a_2^\dagger - a_1^\dagger a_2^\dagger\right]|00\rangle\,,\ \zeta = \zeta_1 + i\zeta_2, \tag{17}$$

where $a_i, a_i^\dagger$ are Bose annihilation and creation operators, respectively, $|00\rangle$ is the two-mode vacuum state. Both $|\eta\rangle$ and $|\zeta\rangle$ possess the orthogonal and completeness relation

$$\int \frac{d^2\eta}{\pi}\,|\eta\rangle\,\langle\eta| = 1,\ \ d^2\eta = d\eta_1 d\eta_2,\ \left\langle\eta'\right|\left.\eta\right\rangle = \pi\delta^{(2)}\left(\eta' - \eta\right),$$

$$\int \frac{d^2\xi}{\pi}\,|\zeta\rangle\,\langle\zeta| = 1,\ \ d^2\xi = d\zeta_1 d\zeta_2,\ \left\langle\zeta'\right|\left.\zeta\right\rangle = \pi\delta^{(2)}\left(\zeta' - \zeta\right). \tag{18}$$

They obey the eigenvalue equations

$$(X_1 - X_2)\,|\eta\rangle = \sqrt{2}\eta_1\,|\eta\rangle\,,\ \ (P_1 + P_2)\,|\eta\rangle = \sqrt{2}\eta_2\,|\eta\rangle\,,$$

$$(X_1 + X_2)\,|\zeta\rangle = \sqrt{2}\zeta_1\,|\zeta\rangle\,,\ \ (P_1 - P_2)\,|\zeta\rangle = \sqrt{2}\zeta_2\,|\zeta\rangle\,. \tag{19}$$

Thus for a bipartite system, $|\eta\rangle$ is the common eigenvector of two particles' relative coordinate and their total momentum with the eigenvalues being the real and imaginary parts of the complex variable η, respectively. It is Einstein *et al.* (1935) who firstly used $[(X_1 - X_2)\,,\ (P_1 + P_2)] = 0$ to challenge the incompleteness of quantum mechanics and consequently show quantum entanglement. The overlap of $\langle\zeta|\ \eta\rangle$ is

$$\langle\zeta|\ \eta\rangle = \frac{1}{2}\exp\left(\frac{\zeta^*\eta - \eta^*\zeta}{2}\right). \tag{20}$$

Calculating the matrix element of F_2 sandwiching between $\langle\eta|$ and $|\zeta\rangle$, using (19) and (Fan, 2003)

$$\exp\left[i\lambda\left(X_1P_2 + X_2P_1\right)\right] = \int \frac{d^2\zeta}{\mu\pi}\,|\zeta/\mu\rangle\,\langle\zeta|$$

$$= \mu\int \frac{d^2\eta}{\pi}\,|\mu\eta\rangle\,\langle\eta| \equiv U_2(\lambda)\,,\ \mu = e^\lambda, \tag{21}$$

we have

$$\begin{aligned}
&\langle\eta|\, F_2\,(A,\,B,\,C)\,|\zeta\rangle \\
&= \exp\left(\frac{iC}{2A}\,|\eta|^2 - \frac{iB}{2A}\,|\zeta|^2\right)\langle\eta|\int\frac{d^2\zeta'}{A\pi}\,\left|\zeta'/A\right\rangle\left\langle\zeta'\right|\,\zeta\rangle \\
&= \frac{1}{A}\exp\left(\frac{iC}{2A}\,|\eta|^2 - \frac{iB}{2A}\,|\zeta|^2\right)\langle\eta|\,\zeta/A\rangle \\
&= \frac{1}{2A}\exp\left(\frac{iC}{2A}\,|\eta|^2 - \frac{iB}{2A}\,|\zeta|^2\right)\exp\left[\frac{1}{2A}\left(\eta^*\zeta - \eta\zeta^*\right)\right]
\end{aligned} \tag{22}$$

It then follows from (18) and (20) that

$$\begin{aligned}
\left\langle\eta'\right|\,F_2\,(A,\,B,\,C)\,|\eta\rangle &= \textstyle\int_{\infty}^{\infty}\frac{d^2\zeta}{\pi}\left\langle\eta'\right|\,F_2\,|\zeta\rangle\,\langle\zeta|\,\eta\rangle \\
&= \tfrac{1}{4A}\exp\left(\tfrac{iC}{2A}\left|\eta'\right|^2\right)\textstyle\int\frac{d^2\zeta}{\pi}\exp\left[-\tfrac{iB}{2A}\,|\zeta|^2 + \tfrac{1}{2A}\left(\eta'^*\zeta - \eta'\zeta^*\right) + \tfrac{1}{2}\left(\zeta^*\eta - \zeta\eta^*\right)\right] \\
&= \tfrac{1}{2iB}\exp\left[\tfrac{i}{2B}\left(A\,|\eta|^2 - i\left(\eta\eta'^* + \eta^*\eta'\right) + D\left|\eta'\right|^2\right)\right] = \mathcal{T}^M\left(\eta',\,\eta\right),
\end{aligned} \tag{23}$$

which is just the transform kernel of a two-dimensional GFT. Letting a state vector $|f\rangle$ undergoes a SU(1,1) squeezing operator transform, $F_2\,(A,\,B,\,C)\,|f\rangle = |g\rangle$, then the corresponding two-dimensional GFT in complex η space can be written as

$$\begin{aligned}
g\left(\eta'\right) &\equiv \left\langle\eta'\right|\,g\rangle = \left\langle\eta'\right|\,F_2\,(A,\,B,\,C)\,|f\rangle \\
&= \int_{\infty}^{\infty}\frac{d^2\eta}{\pi}\left\langle\eta'\right|\,F_2\,(A,\,B,\,C)\,|\eta\rangle\,\langle\eta|\,f\rangle \\
&= \int_{\infty}^{\infty}d^2\eta\,\mathcal{T}^M\left(\eta',\,\eta\right)f\,(\eta) \equiv \left[\mathcal{F}^M f(\eta)\right]\left(\eta'\right),
\end{aligned} \tag{24}$$

where the subscript 2 means two-dimensional. The corresponding multiplication rule is

$$\mathcal{T}^{M''}\left(\eta'',\,\eta\right) = \int_{-\infty}^{\infty}d^2\eta'\,\mathcal{T}^{M'}\left(\eta'',\,\eta'\right)\mathcal{T}^M\left(\eta',\,\eta\right). \tag{25}$$

5. THE SCALING LAW OF TWO-DIMENSIONAL GFT GAINED VIA $|\eta\rangle$ AND SU(1,1) SQUEEZING OPERATOR

A practical optical transform is often dependent on the scale of the original function, so a scaling rule (or similarity theorems) of GFT is useful. Alieva *et al.*, (1996) presented a scaling rule of GFT by integral transform for (1) and (2). Here we show how to directly employ the property of SU(1,1) squeezing operator to derive the scaling rule of GFT in η space. Let $f(\eta) = \langle\eta\,|f\rangle$ be the input light field from an object (state). By considering the scaling of the object itself as an extra GFT $\mathcal{T}^S$ (the first GFT),

$$\left[\mathcal{T}^S f(\eta)\right](\eta') = \mu f(\mu\eta'), \tag{26}$$

where the scaling parameter matrix S is

$$S=\begin{pmatrix} 1/\mu & 0 \\ 0 & \mu \end{pmatrix}. \tag{27}$$

Then taking the light propagation emitted from the scaled object as the second GFT, which is parameterized by $M(z_1) = [A(z_1), B(z_1); C(z_1), D(z_1)]$ (here z_1 is located at the transversal plane of the image), according to the multiplication rule of GFT (25), the total effect can be equal to a GFT of the unscaled object with the parameter matrix M',

$$\begin{pmatrix} A(z_1) & B(z_1) \\ C(z_1) & D(z_1) \end{pmatrix}\begin{pmatrix} 1/\mu & 0 \\ 0 & \mu \end{pmatrix} = \begin{pmatrix} A(z_1)/\mu & \mu B(z_1) \\ C(z_1)/\mu & \mu D(z_1) \end{pmatrix} \equiv M'. \tag{28}$$

Substituting M' into (23) we immediately know the resultant GFT is

$$\begin{aligned}\left[\mathcal{T}^M f(\mu\eta)\right](\eta') &= \tfrac{1}{\mu}\left[\mathcal{T}^M(\mathcal{T}^S f(\eta))\right](\eta') = \tfrac{1}{\mu}\left[\mathcal{T}^{M'} f(\eta)\right](\eta') \\ &= \tfrac{1}{2\pi i\mu B(z_1)}\int_{-\infty}^{\infty} d^2\eta \exp\left[\tfrac{i}{2\mu^2 B(z_1)}\left(A|\eta|^2 - i\mu\left(\eta\eta'^* + \eta^*\eta'\right) + \mu^2 D\left|\eta'\right|^2\right)\right] f(\eta),\end{aligned} \tag{29}$$

note that in (29) the object is still $f(\eta)$, which is similar to (4) in (Alieva and Agullo-Lopez, 1996). The scaling law then can be stated as: the above GFT M' can be equivalent to another GFT $M''(z_2) \equiv [A(z_2), B(z_2); C(z_2), D(z_2)]$ (i.e., results in an image in a different transversal plane z_2) applied to the original (unscaled) object with its output field being a scaled image of order ν at z_2, which we shall regard as another GFT with transforming parameter $S' = [1/\nu, 0; 0, \nu]$. Then using the group multiplication rule we should identify

$$\begin{aligned}\begin{pmatrix} A(z_1)/\mu & \mu B(z_1) \\ C(z_1)/\mu & \mu D(z_1) \end{pmatrix} &= \begin{pmatrix} 1/\nu & 0 \\ 0 & \nu \end{pmatrix}\begin{pmatrix} A(z_2) & B(z_2) \\ C(z_2) & D(z_2) \end{pmatrix} \\ &= \begin{pmatrix} A(z_2)/\nu & B(z_2)/\nu \\ \nu C(z_2) & \nu D(z_2) \end{pmatrix},\end{aligned} \tag{30}$$

from which, we get

$$\nu = \frac{B(z_2)}{B(z_1)\mu}, \tag{31}$$

and

$$\frac{B(z_1)}{B(z_2)} = \frac{A(z_1)}{\mu^2 A(z_2)}, \tag{32}$$

which agree with (Alieva and Agullo-Lopez, 1996). With (31) and (32) being satisfied, (30) can be rewritten as

$$\begin{aligned}\left[\mathcal{T}^{M(z_1)} f(\mu\eta)\right](\eta') &= \frac{1}{\mu}\left[\mathcal{T}^{M(z_1)}(\mathcal{T}^{S} f(\eta))\right](\eta') \\ &= \frac{1}{\mu}\left[\mathcal{T}^{M'} f(\eta)\right](\eta') = \frac{\nu}{\mu}\left[\mathcal{T}^{M(z_2)} f(\eta)\right](\nu\eta') \\ &= \frac{B(z_2)}{\mu B(z_1)}\left[\mathcal{T}^{M(z_2)} f(\eta)\right](\nu\eta'). \end{aligned} \tag{33}$$

It should be noticed that with condition (32), the extra phase factor in (6) of Alieva and Agullo-Lopez (1996) automatically vanishes.

In sum, we have found the appropriate quantum mechanical SU(1,1) squeezing operator which is composed of quadratic combination of canonical operators for both one-mode and two-mode cases. In two-mode case we have combined the two-mode SU(1,1) squeezing operator and the entangled state basis to yield the two-dimensional GFT kernel in η space and the scaling rule. The advantages of SU(1,1) squeezing operator is exhibited and why using the quadratic combination of canonical operators is explained. The usefulness of entangled states is demonstrated again in the link between quantum optics and Fourier optics.

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